

**USING PROBABILITY TO DETERMINE AIR FLOWS FOR FUME
HOOD DESIGN, FOR DUCT SYSTEM SIZING, CHILLER WATER NETWORK
SIZING, AND HVAC CENTRAL PLANT LOADS**

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INTRODUCTION

In developing the application of the theory of probability to the problem of plumbing design loads, Dr. Roy Hunter in 1932 at the National Bureau of Standards first assumed that the operation of the principal fixtures making up the plumbing system could be considered as purely random events. His goal was to quantify on the basis of probability a means of sizing waste conduits based on usage demand at a probable "not to exceed" failure rate. While the use of plumbing fixtures is not entirely a random event, and while problems associated with fume hood usage, duct designs that serve hoods and the load on a central plant attributable to hood operation may not appear to have any relation to plumbing fixture usage, Hunter's basic approach can nevertheless be applied to laboratory air flow and HVAC subsystems for design sizing. Moreover, approximate allowances can be made in some cases for deviation from complete randomness for both the fume hoods and plumbing fixtures which allows extension of the basic theory developed by Dr. Hunter for plumbing fixtures to fume hood usage as it impacts air distribution, exhaust, chilled water and central plant design and system sizing.

BACKGROUND CONSIDERATIONS

The maximum probability of use of a fume hood, how it is determined, and how it affects system design merits some discussion. While a hood may be in continuous use with an experiment on-going within it at all times, most would agree that maximum safe hood operations will always occur when a hood's sash is closed. Laboratory technician simply do not work in front of fume hoods all day long. They set experiments up and check them periodically while they go about doing other task in the laboratory. The conclusion concerning hood sash closure when an operator is not present in front of a hood, is not new and was first espoused by H.W. Alyea, Chief Field Engineer, Johnson Service Co. (now Johnson Controls, Inc.) in 1951 when he stated,

"The amount of air introduced into the laboratory is only that required to maintain desired face velocity through the hood doors. Since the hood doors are kept closed as much as possible for reasons of safety, this results in a considerable savings in the amount of air supplied [for hood make up exhaust], with proportional reduction in cooling demand, and considerable filter life."

This suggests a hood's sash(es) should only be open or partially open when an operator is standing in front of the hood to accommodate operator manipulated activities within the hood. When an operator has a hood open and is manipulating something within the hood this will be defined as "active [hood] use". At all other times, while the hood may be operational with exhaust flow from it, especially where chemical are involved, there should be no reason for the hood sashes to remain open. Hence the term "active use" and its application to fume hoods, particularly when they function in a variable air volume system where exhaust flow is reset by hood sash opening, is of special concern to cost conscious laboratory owners and operators.

Alyea's comments must be interpreted in light of the 1940s Atomic Energy era laboratories he referred to, i.e., those where the flow of air was used only as a means containing fugitive material in a fume hood. These era laboratories were built in locations where outside air was not "conditioned" for occupant comfort, save for some heating of the air. For example, consider location, Los Alamos, NM where the air is always cool and dry, requiring tempering of outside air only for occupant comfort. The outside air flow and fume hood systems that utilized the Weber "constant face velocity" control system "contained" very well because of the draw through concept. No supply air, conditioned or unconditioned, was forced into the building. Powered only by the fume hood exhaust fans, air was drawn into a simple building across a unit heater. It swept the halls as it flowed to the laboratory room, which it entered via a louver in the door or the open door into the lab room. The air then swept through the lab and transported all airborne material toward the operating. Hood face velocity was maintained constant with changing sash position through the employment of a rack and pinion gear, and the action of a cam custom designed for each hood, Weber's concept. The cam operated a damper in the hood exhaust duct. These variable air volume (VAV) systems, which featured consistent flow with varying sash position (called inferential face velocity), performed extremely well in their role of containing materials within the fume hood.

Given the consideration that the primary goal of the laboratories and their air flow and ventilation systems was to create a safe working environment subject to "fail safe" considerations, the primary goals were containment ventilation based on currently meeting the following conditions:

- First, maximum containment of materials in the fume hood under all operating conditions.
- Second, maximum containment of materials in the room that houses the fume hood under all operating conditions.

This was done by design of the air flow system incorporating the following.

A. Maintain fume hood face velocity constant at all openings regardless of static or dynamic hood sash position or airflow control system response dynamics.

b. Always keep the hood sash at its minimum opening.

Energy savings and operating cost avoidance were not considerations at that time. Comfort conditioning of the air stream into the lab was not done and was not needed because of the geographical locations for labs chosen. And the laboratory air flow systems to include the make up air system, fume hood and exhaust trains, performed much better than most ventilation systems in use today.

Given a laboratory air flow system that meets these two primary conditions, with safety best achieved through containment ventilation, the following illustrates how a secondary goal of lower first cost and energy savings must necessarily follow. A simple but straightforward means of evaluating potential energy savings associated with a laboratory air flow system, as it applies to variable flow fume hoods with sash position, always kept at minimum closed position, is presented. It is assumed that the goal of hood operations is to maintain a relatively constant hood capture velocity at all sash openings and that some mechanism exists within or on the hood to accommodate this purpose. Moreover, while it is assumed that each hood operates at constant inferential face velocity achieved by varying exhaust flow, a mechanism must exist whereby hood exhaust flow resets room supply flow. The supply must be offset by a fixed amount less than the exhaust flow to satisfy the intent of NFPA 45, *Fire Protection for Laboratories Using Chemicals*, which requires negative laboratory pressurization at all time be achieved.

USING HUNTER'S PROBABILITY ANALYSIS

We are now able to see how to determine the number m hoods out of a total of n hoods we should assume to be in simultaneous operation for determining the design loads for HVAC systems. Once we have established the value of m , the design hood exhaust air flow load is found by multiplying m by the average rate of demand in cfm of one hood yielding

$$Q_d = mq \quad (1)$$

It is unique, and should be noted that the demand makeup rate of air flow of one hood, q , represents a differential air quantity. This differential air quantity represents the difference between maximum hood ventilation rate less the maximum exhaust flow rate required for comfort conditions. This rate of demand thus assumes the hood is being used for laboratory exhaust in a "once-through" system. If this is not the case and the laboratory ventilation cooling rate and hood exhaust are independent, q then equals the maximum hood exhaust rate. This latter situation would occur when, say, a fan coil unit exists in the room and hood operation is independent of it, a dangerous situation in most cases that while satisfying code requirement, often makes for a homogenous mixture of fugitive gases in the volume of air in a room.

The criterion that will be used for adequate design has already been stated as follows: *The system will be considered to operate satisfactorily if it is so proportioned that it will supply adequately the simultaneous demand*

load for such number m of the n hoods comprising the system that more than m will probably not be found in simultaneous operation more than 1 percent of the time. This condition can be expressed as follows:

$$p_0^n + p_1^n + \cdots + p_{m-1}^n + p_m^n \geq 0.99 \quad (2)$$

where m is the smallest integer for which this relation is true. In this equation, p_0^n represents the probability of finding none of the n hoods in operation, etc. Appendix A presents the mathematical basis for the statistical analysis and the binominal calculation macro in EXCEL can be use to calculate the various probabilities, above, i.e., the $p_0, p_1, p_2 \dots p_m$. The smallest value of m for which equation (2) is true gives the number of hoods for which the system should be designed.

Equation (2) suffices to yield the desired value of m , but the computation is extremely laborious without a computer, and several methods of reducing the labor to a minimum have been developed. Tables are available giving the summation of the remainder of the series of equation (2) for values of n up to 50, a micro can be used or a computer program can be developed to evaluate the magnitude .

$$p_{m+1}^n + p_{m+2}^n + \cdots + p_{n-1}^n + p_n^n \leq 0.01 \quad (3)$$

Equation (3) can also be written

$$\sum_{r=m+1}^{r=n} \binom{n}{r} (1-p)^{n-r} p^r \leq 0.01 \quad (4)$$

which corresponds to the form given in the tables of the binomial probability distribution, except that here the expression $1-p$ replaces the symbol q in the tables.

APPLICATION

Before proceeding with the practical process of determining design loads, we shall compute a few values of the probabilities in the series given by equation (2) for the hypothetical system of 36 ganged fume hoods. While the ganging of 36 or any number of hoods on a common exhaust trunk duct system is arbitrary for this example, this number of hoods on an exhaust duct system merits illustration for two reasons: first this is the number of potential hoods applied to each of 24 separate exhaust systems in Exxon Research and Engineering Company's Clinton Facility that has been in operation for over twenty years, and second, while this analysis and methodology was developed after this project was complete, other system aspects associated with this facility are used in subsequent examples in this paper.

It was assumed that each hood in this system is used with the average frequency of 15%. This gives the elementary probability p of finding a particular hood in operation at any arbitrarily chosen instant of observation of 0.15.

$$p_0^n = \binom{n}{0} (1-p)^{n-0} p^0 = (1-p)^n = (0.85)^{36} = 0.00288 \quad (5)$$

The probability of finding exactly one of the 36 hoods in use is

$$p_1^n = \binom{n}{1} (1-p)^{n-1} p = \frac{n}{1!} (1-p)^{n-1} p = 36(0.85)^{35} = 0.01828 \quad (6)$$

Proceeding in the same way, for computing the probability of finding exactly two hoods in use

$$\begin{aligned} p_2^n &= \binom{n}{2} (1-p)^{n-2} p^2 = \frac{n(n-1)}{2!} (1-p)^{n-2} p^2 \\ &= \frac{36 \times 35}{2} (0.85)^{34} (0.15)^2 = 0.05646 \end{aligned} \quad (7)$$

and for the probability of finding exactly three of the 36 hoods in use

$$P_3^n = \binom{n}{3} (1-p)^{n-3} p^3 = \frac{n(n-1)(n-2)}{3!} (1-p)^{n-3} p^3 \quad (8)$$

$$= \frac{36 \times 35 \times 34}{3 \times 2} (0.85)^{33} (0.15)^3 = 0.1129$$

Proceeding in the same way, we compute the probabilities through P_{10}^n with results given in Table 1.

TABLE 1

Values of the probability of Finding 0, 1, 2, ..., 10 Fume Hoods in Use out of 36 Hoods in Simultaneous Operation at the 15% Frequency of Use

Active Hoods Out of 36	% of Use Out of 36 Hoods	Sum
0	0.00288	0.00288
1	0.01828	0.02116
2	0.05646	0.07762
3	0.11293	0.19055
4	0.16441	0.35496
5	0.18568	0.54064
6	0.16930	0.70994
7	0.12804	0.83799
8	0.08191	0.91990
9	0.04497	0.96487
10	0.02143	0.98629
11	0.00894	0.99523

The column under the heading "% of Use Out of 36 Hoods" represents the probability that r "active hoods", and r only, out of a total of 36 hoods will be found operating at an arbitrarily selected instant of observation. It indicates, as an example, that exactly five hoods active occurs with a maximum probability of 18.568%.

If we sum these probabilities, commencing with P_0^n 0.00288, we find that the smallest number of hoods for which this sum exceeds 0.99 is 11. Hence, we take 11 as the number that will probably not be found in simultaneous use more than 1% of the time, i.e. – "the probability of limited failure". The design load for the main exhaust fan and the duct at the fan of this system is given by equation (1).

In the above, q is the nominal differential flow associated with each hood at its full open sash position. Since one rarely works in front of a hood that is in its full open position, these results are still significantly biased and air load, duct size, and mechanical equipment size can be reduced further if some means of justifying the reduction can be developed. An example of a justifying means of reductions includes installation of automatic sash positioning systems (ASPS) on fume hoods that only open a hood to its half height position rather than the full open position and closes the hood sash regardless of position when the operator walks away from the hood. Use of a properly functioning ASPs applied to a fume hood would essentially reduce the nominal differential flow by 50% in the

analysis that follows because of air flow associated with full sash opening versus reduced air flow corresponding to a normally used automatic positioning to half height position.

CASE STUDY: Exhaust Flow Diversity Applied to Gang Exhaust System Design.

Suppose a system of 36 fume hoods are ganged together and served by the same exhaust fan (see Figure 1). Assume the hoods for analysis purposes are all the same size (if they are of different sizes, this can be treated by a similar but modified methodology). Assume further that comfort cooling in each laboratory requires continuous exhaust of 789 cfm (room) net or 200 cfm per hood if four hoods are installed as a minimum value where the basic laboratory is composed of two modules and a module is 11' wide by 33' long having a total floor area of 363 square feet. If the laboratory space is interior to a building and no significant heat generation equipment exists outside of fume hood, the flow of conditioned air required to offset internal heat gain should be approximately 1.1 cfm/sq. foot. Hence, assume the hoods are in groups of four (4) hoods per lab (i.e. "clustered") in nine two-module labs along a corridor served by a single exhaust fan. Assuming all hoods are the same size, with 750 cfm exhaust from each at full open, the trunk duct and fan would normally be sized at 750 X 36 = 27,000 cfm without diversity.

Now consider that each hood is in 'active use" approximately 15% of any time during the day. If we assume use of a hood is a random event, then we can apply the binomial distribution theorem to their use. We have shown that for 36 hoods in operation 15% of the time each, a maximum of 11 hoods will ever be in simultaneous operation with 99% certainty. This means that the fan and trunk duct can be sized at 13,250 cfm as shown below.

$$36 \text{ taps} \times 200 \text{ cfm per hood tap} + 11 \text{ hoods} \times (750 - 200) \text{ cfm/hood} = 13,250 \text{ cfm (9)}$$

Hence, there is a considerable energy savings associated with sizing a fan system at 13,250 cfm instead of at 27,000 cfm, i.e., VAV system operation vs. constant volume system operation. Moreover, we can also apply this to sizing the trunk duct down the corridor by determining the number of hoods exhausted through each trunk section back to the fan. This is illustrated in Table 2.

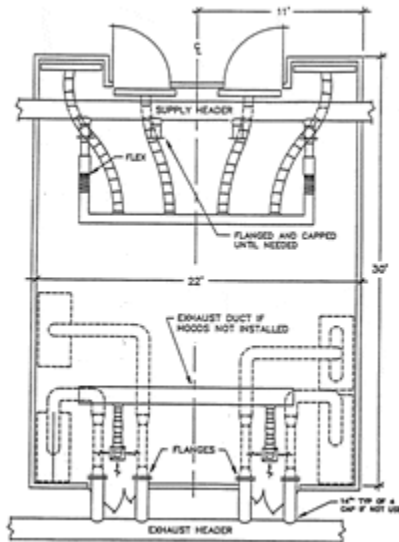
TABLE 2

Maximum Number of Hoods in Simultaneous Operation at 99% Capacity With Progression Toward the Fan and at the 15% Frequency of Use

Total # of Hoods		Cooling Load Ventilation		Sizing #		Differential Flow		Net Section Flow in CFM
4		200	+	3		(750-200)	=	2450
8	x	200	+	4	x	(750-200)	=	3800
12	x	200	+	5	x	(750-200)	=	5150
16	x	200	+	6	x	(750-200)	=	6500
20	x	200	+	7	x	(750-200)	=	7850
24	x	200	+	8	x	(750-200)	=	9200
28	x	200	+	9	x	(750-200)	=	10550
32	x	200	+	10	x	(750-200)	=	11900
36	x	200	+	11	x	(750-200)	=	13250

FIGURE 1

TYPICAL TWO-MODULE LABORATORY PLAN



In all cases the runouts from the hoods to the trunk duct would be sized for 750 cfm. Moreover, progressing from the most fan remote section of the duct toward the fan, calculations indicate that at the 15% active use level, less than three of four hoods would be in use in the first cluster, given a "limited failure" not to exceed 1% of the time. This number is calculated using equation (4) with $n = 4$, concluded with completion of a table similar to that illustrated by Table 1. For the next duct section, four of eight taps serving the two most remote clusters would be in cumulative operation progressively toward the fan with a failure level not to exceed 1% as predicted again by equation (4); 5 of 12 would be in use in the next trunk section, and so forth. With this criteria the calculated flow in each section in the trunk duct after each "cluster" joins the trunk is illustrated under the heading labeled "Net Section Flow in CFM" in Table 2.

While the above analysis has been applied to an exhaust system, the implications associated with it also affect supply side design considerations. If supply side air flow into a two-module laboratory is controlled via the amount of flow being exhausted from the space through hoods serving it, then the same argument applies to sizing the supply air duct system. As an example, with reference to Figure 1, if supply side air flow were to equal exhaust flow, the normal supply flow basis for sizing AHU #1 without diversity would be the 27,000 cfm for all hoods at full operation. If we apply the concept of limited failure at the 1% level to the supply and to exhaust, duct and fan distribution systems, we would only have 11 hoods operating at a time and the maximum air handler system airflow would be 13,250 cfm.

CASE STUDY: Diversity Application to Chilled Water Flow at Coils.

Since supply side air systems in large laboratory complexes are logically served by chilled water systems, the analysis also has significance to chilled water pipe sizing design and to the flow rates needed to serve laboratory air handlers. This is discussed as follows.

If chilled water from a plant is piped so that one chilled water header supplies AHU 1, 2, and 3, as illustrated in Figure 2, heat gain from air flow through each respective AHU could be calculated as follows:

TABLE 3

Maximum Number of Hoods in Simultaneous Operation
at 99% Capacity Served by a Common Single Chilled
Water Supply and at 15% Frequency of Use

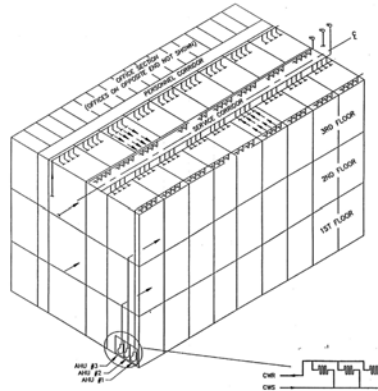
AHU #1 only	13,250 cfm with 11 of 36 hoods operating
AHU #'s 1 & 2 only	24,300 cfm with 18 of 72 hoods operating
AHU #'s 1, 2 & 3	35,350 cfm with 25 of 108 hoods operating

Since the probability of use associated with any hood is 15%, equation (4) applied to all 108 hoods served by a common chilled water pipe simultaneously serves the three air handlers. The lateral pipe from the main would be of a common size as would inter-connecting pipe between airhandlers; its size would be based on peak thermal demand necessary to condition 35,350 cfm of outside air.

If the piping arrangement were different and AHU #'s 1 & 2 were served by common supply with AHU #3 served by a separate supply, the supply piping sizes would be based on outside air flow of 24,300 and 13,250 cfm respectively as illustrated in Table 3.

FIGURE 2

AIR DISTRIBUTION PLAN FOR THREE STORY, 54-TWO MODULE LABORATORY COMPLEX



CASE STUDY: Diversity Application to Chiller and Hot Water Plant Design.

Reduced building exhaust and companion supply air flow translates into less chilled or hot water flow, less chiller and boiler capacity, and reduced chilled water, condenser water and cooling towers requirements. As a concluding example, consider the probability analysis applied to Exxon Research and Engineering's Clinton Facility with distribution systems partially illustrated by Figures 1 and 2 and assume that:

1. Variable air volume hood operation exists as a function of hood sash position on all hoods with sashes guaranteed closed when the hood is not in "active use";
2. Hood usage level is 15%, a randomly occurring event, and each hood is sized for 750 cfm exhaust corresponding to *full sash opening* with no allowance for reduced air flow at partial hood sash opening. A more optimal form of control is now available that yields constant inferential fume hood face velocity, variable exhaust flow control as a function of fume hood sash opening, complete with ASPS;
3. Thirty-six (36) hoods exist on any single exhaust system, four hoods to a two module laboratory, and there are twenty-four (24) exhaust systems serving the building to yield a building potential total of 864 hoods; all of which are assumed installed and functioning;
4. Space cooling requirements dictate that 200 cfm per hood is needed for maximum cooling of the laboratory spaces.

Given the above, air flow and chilled water piping for the laboratory section of the building would never exceed

$$864 \text{ taps} \times 200 \text{ cfm/hood tap} + 155 \text{ hoods} (750 - 200) \text{ cfm/hood} = 258,050 \text{ cfm} (10)$$

and diversity at the 1% failure level would be 37% based on 258,050 cfm vs. 648,000 cfm (i.e. - 864 hoods X 750 cfm/hood).

Finally, if certainty level is increased from 99 to 99.99%, *only 15 more out of 864 hoods would be active and chiller plant diversity would be increased from 37% at the 1% failure level to 39.1% at a 0.01% failure level!*

CONCLUSIONS

Containment ventilation achieved by precise and accurate control of fume hood exhaust with sash position, i.e., "inferential face velocity control", coupled with keeping the hood sashes closed when not in "active use" allows a laboratory air flow system to achieve maximum containment ventilation performance.

The above analysis procedures and examples illustrate the tremendous potential energy and operational cost avoidance savings that have driven the development of variable air volume system applications to laboratories. The analysis also provided a methodology that can be used to guarantee savings results if required control system

attributes exist and are made to function correctly.

The variable air volume control technological development has been on-going since 1943 when Weber successfully developed his variable air volume laboratory fume hood exhaust system. It focused, with primary concern, on enhancement of containment ventilation safety. And while the technology development efforts have strayed away from the goal of containment ventilation safety in favor of energy savings, only within the last few years has the application technology been refocused on the greater primary goal of containment ventilation safety. This has been done by refining and redefining control goals in order to produce commercially available systems that are simple to install, reliable to operating, easy to maintain and that again focus on maximizing containment ventilation safety.

Refocus on containment ventilation has largely evolved only within the last few years after it became clear that psychologically based means that "encourage" operators to close fume hoods when the hoods are not in active use do not work. Closing fume hood sashes must be accomplished by some mechanical control means if it is guaranteed to be done. Thus, while the commercial technology has matured and has recently begun to be proven, the net result is that only now does opportunity exist to users for achieving these operational cost avoidance savings without potential of jeopardizing safe containment operations in laboratory fume hoods at all time.

Hardware applications that differ by vendor design have now matured to the point where user consideration should warrant serious and objective review of available offerings. Objective user considerations dictate that users and engineers that specify systems understand the application constraints and system limitations associated with vendor offering. These applications constraints include proven consistent operation, precision and accuracy of flow control with hood sash position and consideration of reliability and maintainability differences associated with different vendor system offerings.

Finally, a freeware computer program, *Qloads*, has been developed and is available to the general public for evaluation of energy savings opportunities associated with probability of active use of fume hoods. Energy savings calculations are made on a hour by hour basis using site specific weather data. The energy savings magnitude is then converted to an operating cost on the basis of user supplied utility cost information. This program allows convenient calculation of operating cost versus different fume hood use profiles against utility costing information for evaluation of simple payback and other savings potentials.

REFERENCES

1. Manas, Vincent T., ed., "Design Loads for Plumbing Systems," Chapter 24, National Plumbing Code Handbook, New York: McGraw-Hill Book Company, Inc., 1957; pp. 24-1 to 24-30.
2. Bryant, R. K., Senior Project Engineer (Retired), Plant Operations Division, and Pfister, Hans, Section Head, Facility Engineering Division, Exxon Research and Engineering Company, Clinton Township, NJ, private communication.
3. Fuller, F. H., Senior Consultant (Retired), Engineering Department, E. I. Dupont de Nemours & Company, Wilmington, DE, private communication.
4. Hines, J. D., Project Engineer, Westhollow Research Center, Shell Development Company, Houston, Texas.
5. Tables of Binomial Probability Distribution, National Bureau of Standards, Applied Mathematics, Series 6, 1950.
6. Rudoy, W., Project Director, Cooling and Heating Load Calculation Manual, ASHRAE GRP 158, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., 1978, 345 East 47th Street, New York, NY 10017.
7. The QLOADS program, along with a library of site-specific weather data files, can be downloaded at <http://www.accuaire.com>.
8. Control Systems offering by Accu*Aire Controls, Inc. to include Accu*Aire's sPs or sSa features.

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Appendix A

PROBABILITY ANALYSIS

We define a simple system, consistent with Hunter's definition, as one which consists of fume hoods of a single kind whereby the exhaust of each is individually reset by hood sash position to achieve some means of controlling hood face velocity constant or nearing constant as a function of hood sash opening. Also like Hunter, let us assume there is a large number n of the same size fume hoods and for illustrative purposes, we will assume they are served by a "ganged" exhaust system. Hunter's analysis was concerned with the time T in minutes, on the average, between successive uses of each individual fixture, with t being the duration in minutes associated with hood being in active use. Continuing the analysis as Hunter did, then the probability p that *one particular hood will be used* at any arbitrarily chosen instant of observation of the system would be

$$p = \frac{t}{T} \quad (\text{A-1})$$

And, the probability that *this particular hood* (or any other particular hood) *will not be in use* would be

$$1 - p = 1 - \frac{t}{T} \quad (\text{A-2})$$

The probability of active use associated with fume hoods and the determination and application of a probability value for hood use for a facility must differ from Hunter's application. If one were to be consistent with Hunter's approach, under ideal circumstances one would instrument fume hoods to detect when someone was in front of the hood, when the sash was open, and at what level it was open (i.e. – percent of full value). Unfortunately this data is expensive to obtain, may be specific to a particular location by laboratory type, type of hood, etc. In contrast to Hunter's approach, consider how a maximum upper bound probability of active use can be established and applied to fume hoods as follows.

If one walks through a laboratory building, at any instant of time one will observe a certain percentage of hoods in "active use", i.e., with an operator in front of the hood manipulating something within the hood that requires the sash(es) to be open. The percent of hoods in "active use" at any one time out of the total number of hoods installed as a part of a system represents the probability of use of hoods for this discussion. If random sampling continues and is recorded as a percent for each observation, the maximum probability of active use is selected from all observation data sets taken.

While the peak probability of use may vary by facility type (i.e. – wet chemistry laboratory in a refinery quality control application vs. a medical test laboratory), fume hood type (i.e. – hood sash arrangement that impacts on design flow rate required), usage or service and perhaps other factors, let us assume that it is found to be 5%. If a safety factor of 3 is applied to this probability, an ultra conservative estimate of probability of use of 15% *should never occur*.

The question now remains, how can Hunter's basic approach be extended to determine fume loads, duct and central plant size loads? To illustrate this probabilistic approach consider the following hypothetical situation that is presented and discussed to illustrate the basic methodology. Expansion of the basic concept to a general situation follows.

Consider a large research laboratory dedicated to fundamental research or product development. Assume further that a section in the laboratory occupancy is always at full capacity and can house 36 patients and that as one randomly walks through the section observing all outlets, one might observe that the probability of simultaneous use of all identical fixtures in identical service is normally distributed ranging from 3 to 5%. Consider also that a 3:1 safety factor is applied to yield a maximum upper bound probability of active use as 15%. This yields the follows very conservative analysis basis where

$$p = 0.15 \text{ and } 1 - p = 1 - 0.15 = 0.85 \quad (\text{A-3})$$

Note that what the other $n - 1$ hoods may be doing at the instant when the system is observed has nothing to do with the probabilities give by Eqs. (A-1) and (A-2).

We next determine the probability that *two particular hoods will be found in use* at any arbitrarily chosen instant of observation, disregarding what the other $n - 2$ hoods may be doing at this instant. We have already seen that the probability of finding the first of these two selected hoods in operation is p . Likewise, the probability of finding the

second of these two selected hoods operating is p . Then the probability that both of these two particular hoods will be found in use is p^2 , by the law of compound events. This numerically is

$$p^2 = (0.15)^2 = 0.0225 \quad (\text{A-4})$$

Similarly, the probability of finding *three particular hoods in use* is $p^3 = (0.15)^3 = 0.0034$. And the probability of finding all the n hoods in use is $(0.15)^n$.

We next consider the probability that *two particular fume hoods, but none of the other $n - 2$ hoods will be found in use* at the arbitrarily chosen instant of observation.

Probability of finding the first hood in use p

Probability of finding the second hood in use p

Probability of finding the third hood not in use $1 - p$

Probability of finding the fourth hood not in use $1 - p$

Probability of finding the fifth hood not in use $1 - p$

Probability of finding the n th hood not in use $1 - p$

The probability of this compound event being observed at the chosen instant of observation is the product of the foregoing probabilities, or

$$P = (1 - p)^{n-2} p^2 \quad (\text{A-5})$$

For fume hoods, if $n = 5$, we have this case

$$(1 - p)^{n-2} p^2 = (1 - 0.15)^3 (0.15)^2 = 0.0138 \quad (\text{A-6})$$

We can now pass to the more general case in which *any two* of the n hoods, but none of the other $n - 2$ hoods are found in use at the arbitrarily chosen instant of observation. It has already been shown that the probability of finding *two particular hoods*, but none of the other $n - 2$ hoods in use is $(1 - p)^{n-2} p^2$.

There are as many ways of selecting two hoods out of n hoods as there are combinations of n things taken two at a time. In addition, in the general case, we are interested in determining how many ways of selecting r things from a total of n things there are. Any book on probability will give the expression for this, which is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (\text{A-7})$$

where $\binom{n}{r}$ is the symbol for n things taken r at a time and ! is the symbol for "factorial." To make this concrete, if $n = 5$ and $r = 2$,

$$\binom{5}{2} = \binom{5}{2} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1)(3 \times 2 \times 1)} = 10 \quad (\text{A-8})$$

Thus, if $n = 5$ and $r = 2$, the probability that *any two of the five, but none of the other three, hoods will be found in use* at an arbitrarily chosen instant of observation is

$$10(0.85)^3 (0.15)^2 = 0.138 \quad (\text{A-9})$$

We can now write a general expression for the probability that *any r hoods, and r only, out of a total of n hoods will be found operating* at an arbitrarily selected instant of observation:

$$P_r^n = \binom{n}{r} (1-p)^{n-r} p^r \quad . \text{ (A-10)}$$

When we observe the system, it is certain that we shall find some number r of n hoods in operation, where r may have any integral value from 0 to n . In the theory of probability, certainty is represented by the number unity. Hence if we sum all the probabilities represented by equation (A-10), which is the probability of one particular even out of those just mentioned, we get the relation

$$P_r^n = \sum_{r=0}^n \binom{n}{r} (1-p)^{n-r} p^r = 1 \quad . \text{ (A-11)}$$

We note also that equation (A-10) represents one term of and equation (A-11) represent the entire binomial expansion of $[p + (1-p)]^n$, as can be found from any text on algebra. Thus the distribution with which we have to do in this problem is of the binomial-expansion type.